Grant element calculation: Formulas

These formulas derive grant element, a measure of concessionality of a loan. Grant element is defined as the difference between its face value and the sum of the present value of debt service to be made by the borrower, expressed as a percentage of the face value of the loan. The assumptions are that the loan amount is fully disbursed and there are no fees other than interest payments.

<table>
<thead>
<tr>
<th>I. Equal principal repayment</th>
<th>II. Lump sum debt service</th>
<th>III. Annuity</th>
</tr>
</thead>
</table>
| \[
\left(1 - \frac{r}{n}\right) * \left[1 - \frac{1}{(1 + d)^{n/g} - (1 + d)^{(n*w)}}\right]
\]
, where
\[
d = (1 + D)^\frac{1}{n} - 1
\]

\[
1 - \frac{1 + r * m}{(1 + D)m}
\]

\[
PV_g = r * \frac{d}{n * d}
\]
d = \((1 + D)^n - 1\), and \(d_g = (1 + D)^g\)

\[
PV_N = \left(\frac{r}{n}\right) * \left[\frac{1}{\left(1 + \frac{r}{n}\right)^n - 1}\right] * \left(\frac{1}{d_g}\right) * \left(\frac{1 - \frac{1}{d_p}}{d}\right),
\]
where
\[
d_p = (1 + D)^p
\]

**Variable**
- \(r\): Interest rate
- \(m\): Maturity (year)
- \(n\): Grace period (year)
- \(n\): Number of repayments per annum
- \(D\): Discount Rate of 5 percent
- \(p\): Principal repayment period(s) (year) (m-g)
- \(N\): Total number of repayments (p*n)
- \(g\): Interval (year)